Taylor's Theorem

Taylor's Theorem is a fundamental result in calculus that allows us to approximate a function near a given point using polynomials. If a function f(x) is sufficiently smooth (that is, it has derivatives of all required orders) near a point a, then the function can be expressed as a sum of its derivatives evaluated at that point. Mathematically, the Taylor series of a function f(x) about x = a is: $f(x) = f(a) + f'(a)(x - a) + f''(a)/2! (x - a)^2 + f'''(a)/3! (x - a)^3 + ...$ This polynomial approximation becomes more accurate as more terms are included, especially when x is close to a. **Example:**
Find the Taylor series of $f(x) = e^x$ about x = 0 (called the Maclaurin series). First, compute the derivatives of f(x): $f(x) = e^x$, $f'(x) = e^x$, $f''(x) = e^x$, and so on. Now evaluate these derivatives at x = 0: f(0) = 1, f'(0) = 1, f''(0) = 1, f'''(0) = 1. Substitute into the Taylor formula: $e^x = 1 + x + x^2/2! + x^3/3! + ...$ Thus, the exponential function can be approximated near x = 0 using this infinite polynomial. Taylor's Theorem is widely used in science and engineering for approximation and numerical analysis.